RECOGNISING ACHIEVEMENT

**ADVANCED SUBSIDIARY GCE**

# MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

**QUESTION PAPER**

Candidates answer on the printed answer book.

**OCR supplied materials:**
- Printed answer book 4755
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**
- Scientific or graphical calculator

**Wednesday 19 January 2011**
**Afternoon**

**Duration:** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER / INVIGILATOR**

- Do not send this question paper for marking; it should be retained in the centre or destroyed.





**Turn over**

**Section A** (36 marks)

**1** Find the values of $P$, $Q$, $R$ and $S$ in the identity $3x^3 + 18x^2 + Px + 31 \equiv Q(x+R)^3 + S$. **[5]**

**2** You are given that $\mathbf{M} = \begin{pmatrix} 4 & 0 \\ -1 & 3 \end{pmatrix}$.

**(i)** The transformation associated with $\mathbf{M}$ is applied to a figure of area 3 square units. Find the area of the transformed figure. **[2]**

**(ii)** Find $\mathbf{M}^{-1}$ and $\det \mathbf{M}^{-1}$. **[3]**

**(iii)** Explain the significance of $\det \mathbf{M} \times \det \mathbf{M}^{-1}$ in terms of transformations. **[2]**

**3** The roots of the cubic equation $x^3 - 4x^2 + 8x + 3 = 0$ are $\alpha$, $\beta$ and $\gamma$.

Find a cubic equation whose roots are $2\alpha - 1$, $2\beta - 1$ and $2\gamma - 1$. **[7]**

**4** Represent on an Argand diagram the region defined by $2 < |z - (3 + 2\mathrm{j})| \leqslant 3$. **[6]**

**5** Use standard series formulae to show that $\sum_{r=1}^{n} r^2(3 - 4r) = \frac{1}{2}n(n+1)(1-2n^2)$. **[5]**

**6** A sequence is defined by $u_1 = 5$ and $u_{n+1} = u_n + 2^{n+1}$. Prove by induction that $u_n = 2^{n+1} + 1$. **[6]**

**Section B** (36 marks)

**7** Fig. 7 shows part of the curve with equation $y = \dfrac{x+5}{(2x-5)(3x+8)}$.

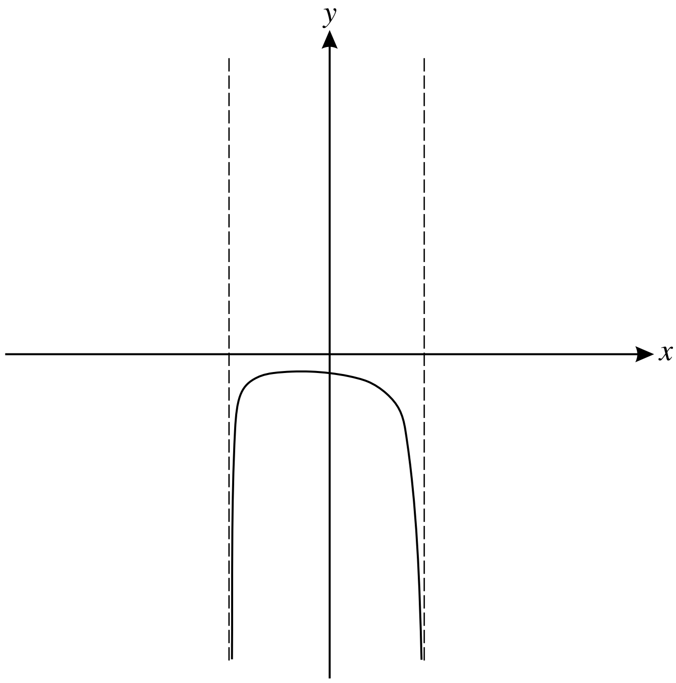


**Fig. 7**

**(i)** Write down the coordinates of the two points where the curve crosses the axes. **[2]**

**(ii)** Write down the equations of the two vertical asymptotes and the one horizontal asymptote. **[3]**

**(iii)** Determine how the curve approaches the horizontal asymptote for large positive and large negative values of $x$. **[3]**

**(iv)** On the copy of Fig. 7, sketch the rest of the curve. **[2]**

**(v)** Solve the inequality $\dfrac{x+5}{(2x-5)(3x+8)} < 0$. **[2]**

**8** The function $f(z) = z^4 - z^3 + az^2 + bz + c$ has real coefficients. The equation $f(z) = 0$ has roots $\alpha$, $\beta$, $\gamma$ and $\delta$ where $\alpha = 1$ and $\beta = 1 + j$.

**(i)** Write down the other complex root and explain why the equation must have a second real root. **[2]**

**(ii)** Write down the value of $\alpha + \beta + \gamma + \delta$ and find the second real root. **[3]**

**(iii)** Find the values of $a$, $b$ and $c$. **[5]**

**(iv)** Write down $f(-z)$ and the roots of $f(-z) = 0$. **[2]**

**9** You are given that $\mathbf{A} = \begin{pmatrix} -2 & 1 & -5 \\ 3 & a & 1 \\ 1 & -1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2a+1 & 3 & 1+5a \\ -5 & 1 & -13 \\ -3-a & -1 & -2a-3 \end{pmatrix}$.

**(i)** Show that $\mathbf{AB} = (8+a)\mathbf{I}$. **[3]**

**(ii)** State the value of $a$ for which $\mathbf{A}^{-1}$ does not exist. Write down $\mathbf{A}^{-1}$ in terms of $a$, when $\mathbf{A}^{-1}$ exists. **[3]**

**(iii)** Use $\mathbf{A}^{-1}$ to solve the following simultaneous equations. **[5]**

$$\begin{aligned} -2x + y - 5z &= -55 \\ 3x + 4y + z &= -9 \\ x - y + 2z &= 26 \end{aligned}$$

**(iv)** What can you say about the solutions of the following simultaneous equations? **[1]**

$$\begin{aligned} -2x + y - 5z &= p \\ 3x - 8y + z &= q \\ x - y + 2z &= r \end{aligned}$$